AP Calculus Pre-Requisite Skills Packet

For all students enrolled in AP Calculus course for the 2023-24 school.

In order to be successful in an AP Calculus course, a student needs to have a firm grasp of the material covered in Algebra I, Algebra II, Geometry, and Precalculus. This packet is an attempt to highlight the skills obtained in those courses that are used most often in an AP Calculus course. Some of the questions require you to use critical thinking on top of the fundamental skills you learned before. During the school year, other skills that may not be covered in this packet will be necessary for success in an AP Calculus class, but this packet should serve as a guide to the skills considered most vital as well as the rigor you should expect from the course.

This packet is not required, but it is ***strongly recommended***. It will give you an opportunity to self-assess how prepared you are for an AP Calculus. If you find yourself struggling with the concepts, go back to your notes from math classes you took in the past, study with a textbook and/or an online resource, ask someone for help, then try the packet again. Know that your success in AP Calculus depends on your understanding and mastery of the concepts and skills covered and your ability to use them in new contexts. Know also that there will be no in-class review of these topics as we will dive into the Calculus right away. There is, however, likely to be a quiz during the first or second week of the school year which will be based solely on the concepts covered in this packet. Students will have the opportunity to ask questions before they are given the quiz.

IMPORTANT:

1. Absolutely NO CALCULATOR or electronic device is to be used on the ENTIRE packet. If there are questions you cannot do without calculator, it indicates that you have to work on those fundamental skills during summer vacation.
2. Although only answers are attached, as you work on this packet SHOW organized WORK WITH DETAILED STEPS. Most incoming AP Calculus students find it hard to communicate their results and solutions and justify their reasoning using proper language, notation and mathematical conventions. Know that these are the skills on which you will be assessed constantly. Like any other skill, they must be learned and practiced so start now!

Have a great summer! We are looking forward to meeting you in August!

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1 Simplify (a) $\dfrac{x^3-9x}{x^2-7x+12}$ (b) $\dfrac{x^2-2x-8}{x^3+x^2-2x}$ (c) $\dfrac{\frac{1}{x}-\frac{1}{5}}{\frac{1}{x^2}-\frac{1}{25}}$ (d) $\dfrac{9-x^{-2}}{3+x^{-1}}$

2 Rationalize the denominator (a) $\dfrac{2}{\sqrt{3}+\sqrt{2}}$ (b) $\dfrac{4}{1-\sqrt{5}}$ (c) $\dfrac{1}{1+\sqrt{3}-\sqrt{5}}$

3 Write each of the following expressions in the form ca^pb^q where c, p and q are numbers

(a) $\dfrac{(2a^2)^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\dfrac{a(2/b)}{3/a}$ (d) $\dfrac{ab-a}{b^2-b}$ (e) $\dfrac{a^{-1}}{(b^{-1})\sqrt{a}}$ (f) $\left(\dfrac{a^{2/3}}{b^{1/2}}\right)^2\left(\dfrac{b^{3/2}}{a^{1/2}}\right)$

4 Solve for x (do not use a calculator)

(a) $5^{(x+1)}=25$ (b) $\dfrac{1}{3}=3^{2x+2}$ (c) $\log_2 x=3$ (d) $\log_3 x^2=2\log_3 4-4\log_3 5$

5 Simplify (a) $\log_2 5+\log_2(x^2-1)-\log_2(x-1)$ (b) $2\log_4 9-\log_2 3$ (c) $3^{2\log_3 5}$

6 Simplify (a) $\log_{10}\left(10^{1/2}\right)$ (b) $\log_{10}\left(\frac{1}{10^x}\right)$ (c) $2\log_{10}\sqrt{x}+3\log_{10}x^{1/3}$

7 Solve the following equations for the indicated variables

(a) $\dfrac{x}{a}+\dfrac{y}{b}+\dfrac{z}{c}=1$, for a (b) $V=2(ab+bc+ca)$, for a

(c) $A=2\pi r^2+2\pi rh$, for positive r (d) $A=P+nrP$, for P

(e) $2x-2yd=y+xd$, for d (f) $\dfrac{2x}{4\pi}+\dfrac{1-x}{2}=0$, for x

8 For the equations (a) $y=x^2+4x+3$ (b) $3x^2+3x+2y=0$ (c) $9y^2-6y-9-x=0$
complete the square and reduce to one of the standard forms $y-b=A(x-a)^2$ or $x-a=A(y-b)^2$

9 Factor completely (a) x^6-16x^4 (b) $4x^3-8x^2-25x+50$ (c) $8x^3+27$ (d) x^4-1

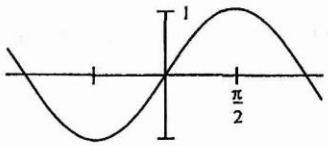
10 Find all real solutions to (a) $x^6-16x^4=0$ (b) $4x^3-8x^2-25x+50=0$ (c) $8x^3+27=0$

11 Solve for x (a) $3\sin^2 x=\cos^2 x$, $0\le x<2\pi$ (b) $\cos^2 x-\sin^2 x=\sin x$, $-\pi<x\le\pi$
(c) $\tan x+\sec x=2\cos x$, $-\infty<x<\infty$

12 Without using a calculator, evaluate the following

(a) $\cos 210^\circ$ (b) $\sin\dfrac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$

(e) $\cos\dfrac{9\pi}{4}$ (f) $\sin^{-1}\dfrac{\sqrt{3}}{2}$ (g) $\tan\dfrac{7\pi}{6}$ (h) $\cos^{-1}(-1)$



13 Given the graph of $\sin x$, sketch the graphs of

(a) $\sin\left(x-\dfrac{\pi}{4}\right)$ (b) $\sin\left(\dfrac{x}{2}\right)$ (c) $2\sin x$ (d) $\cos x$ (e) $\dfrac{1}{\sin x}$

14 Solve the equations (a) $4x^2+12x+3=0$ (b) $2x+1=\dfrac{5}{x+2}$ (c) $\dfrac{x+1}{x}-\dfrac{x}{x+1}=0$

15 Find the remainders on division of

(a) $x^5-4x^4+x^3-7x+1$ by $x+2$ (b) $x^5-x^4+x^3+2x^2-x+4$ by x^3+1

16 (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$ Find all other solutions
(b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$ (All solutions are rational and between ± 1)

17 Solve the inequalities (a) $x^2 + 2x - 3 \leq 0$ (b) $\dfrac{2x-1}{3x-2} \leq 1$ (c) $x^2 + x + 1 > 0$

18 Solve for x (a) $|-x+4| \leq 1$ (b) $|5x-2| = 8$ (c) $|2x+1| = x+3$

19 Determine the equations of the following lines (a) the line through (-1,3) and (2,-4),
(b) the line through $(-1, 2)$ and perpendicular to the line $2x - 3y + 5 = 0$,
(c) the line through $(2, 3)$ and the midpoint of the line segment from $(-1, 4)$ to $(3, 2)$

20 (a) Find the point of intersection of the lines $3x - y - 7 = 0$ and $x + 5y + 3 = 0$
(b) Shade the region in the $x - y$ plane that is described by the inequalities $\begin{cases} 3x - y - 7 & < & 0 \\ x + 5y + 3 & \geq & 0 \end{cases}$

21 Find the equations of the following circles
(a) the circle with centre at $(1, 2)$ that passes through the point $(-2, -1)$,
(b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the $x-$ and $y-$ axes, respectively

22 For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find
(a) the centre and radius, (b) the equation of the tangent at $(-2, 5)$

23 A circle is tangent to the $y-$axis at $y = 3$ and has one $x-$intercept at $x = 1$
(a) Determine the other $x-$intercept (b) Deduce the equation of the circle

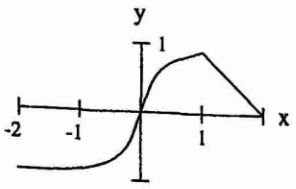
24 A curve is traced by a point $P(x, y)$ which moves such that its distance from the point $A(-1, 1)$ is three times its distance from the point $B(2, -1)$ Determine the equation of the curve

25 (a) Find the domain of the function $f(x) = \dfrac{3x+1}{\sqrt{x^2 + x - 2}}$
(b) Find the domain and range of the functions i) $f(x) = 7$ ii) $g(x) = \dfrac{5x-3}{2x+1}$

26 Let $f(x) = \dfrac{|x|}{x}$ Show that $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$ Find the domain and range of $f(x)$

27 Simplify $\dfrac{f(x+h) - f(x)}{h}$, where (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{1}{x+1}$ (c) $f(x) = x^2$

28 The graph of the function $y = f(x)$ is given as follows



Determine the graphs of the functions
(a) $f(x+1)$ (b) $f(-x)$ (c) $|f(x)|$ (d) $f(|x|)$

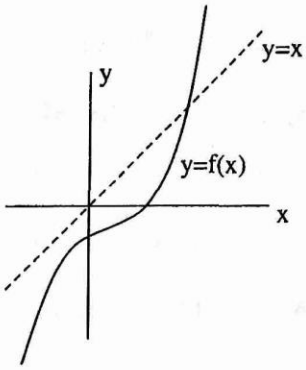
29 Sketch the graphs of the functions (a) $g(x) = |3x + 2|$ (b) $h(x) = |x(x-1)|$

30 (a) The graph of a quadratic function (a parabola) has $x-$intercepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function
(b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$.

31 Write as a single equation in x and y (a) $\begin{cases} x = t+1 \\ y = t^2 - t \end{cases}$ (b) $\begin{cases} x = \sqrt[3]{t} - 1 \\ y = t^2 - t \end{cases}$ (c) $\begin{cases} x = \sin t \\ y = \cos t \end{cases}$

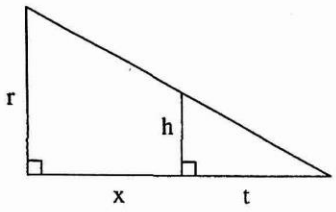
32 Find the inverse of the functions (a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{x+2}{5x-1}$

(c) $f(x) = x^2 + 2x - 1\,,\ x > 0$

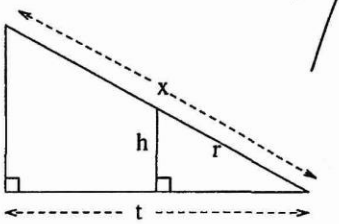


33 A function $f(x)$ has the graph to the right
Sketch the graph of the inverse function $f^{-1}(x)$

34 Express x in terms of the other variables in the picture

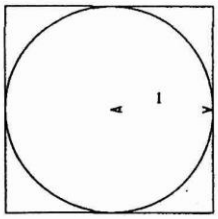


(a)

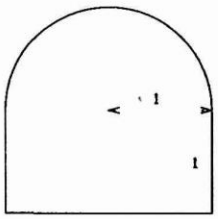


(b)

35 (a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below



(a)



(b)

(b) Find a formula for the perimeter of a window of the shape in the picture (b) above

(c) A water tank has the shape of a cone (like an ice cream cone without ice cream) The tank is 10m high and has a radius of 3m at the top If the water is 5m deep (in the middle) what is the surface area of the top of the water?

(d) Two cars start moving from the same point One travels south at 100km/hour, the other west at 50 km/hour How far apart are they two hours later?

(e) A kite is 100m above the ground If there are 200m of string out, what is the angle between the string and the horizontal (Assume that the string is perfectly straight)

36 You should know the following trigonometric identities

(A) $\sin(-x) = -\sin x$ (C) $\cos(x+y) = \cos x \cos y - \sin x \sin y$

(B) $\cos(-x) = \cos x$ (D) $\sin(x+y) = \sin x \cos y + \cos x \sin y$

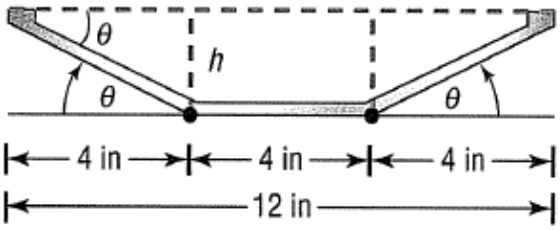
Use these to derive the following important identities, which you should also know

(a) $\sin^2 x + \cos^2 x \equiv 1$ (use C and $\cos 0 = 1$) (b) $\sin 2x \equiv 2 \sin x \cos x$ (c) $\cos 2x \equiv \cos^2 x - \sin^2 x$

(d) $\cos 2x \equiv 2\cos^2 x - 1$ (e) $\cos 2x \equiv 1 - 2\sin^2 x$ (f) $\left|\cos \dfrac{x}{2}\right| \equiv \sqrt{\dfrac{1+\cos x}{2}}$ (g) $\left|\sin \dfrac{x}{2}\right| \equiv \sqrt{\dfrac{1-\cos x}{2}}$

FINAL QUESTION: Constructing a Rain Gutter

A rain gutter is to be constructed of aluminum sheets 12 in wide. After marking off length of 4in from each edge, this length is bent up at an angle θ.



a. Show that the area A of the opening or cross-section (as shown on the illustration) of the gutter as a function of θ is given by $A(\theta) = 16\sin\theta\,(\cos\theta + 1),\ 0 < \theta < \frac{\pi}{2}$..

b. In calculus, you will learn how to find the angle θ that maximizes A. In one of the steps of such optimization you will have to solve the equation $\cos(2\theta) + \cos\theta = 0$.

i. Solve this equation for θ by using the double-angle identity.

ii. Now solve the equation for θ by using the sum of the two cosines identity.

c. What is the maximum area A of the opening?

Answers

1 (a) $\dfrac{x^2+3x}{x-4}$ (b) $\dfrac{x-4}{x^2-x}$ (c) $\dfrac{5x}{x+5}$ (d) $\dfrac{3x-1}{x}$

2 (a) $2(\sqrt{3}-\sqrt{2})$ (b) $-1-\sqrt{5}$ (c) $\dfrac{7+3\sqrt{3}+\sqrt{5}+2\sqrt{15}}{11}$

3 (a) $8a^6\,b^{-1}$ (b) $3a^{\frac{1}{2}}\,b^{\frac{3}{2}}$ (c) $\frac{2}{3}a^2\,b^{-1}$ (d) ab^{-1} (e) $a^{-\frac{3}{2}}\,b$ (f) $a^{\frac{5}{6}}\,b^{\frac{1}{2}}$

4 (a) 1 (b) $-\frac{3}{2}$ (c) 8 (d) $\pm\frac{4}{25}$

5 (a) $\log_2(5(x+1))$ (b) $\log_2 3$ (c) 25

6 (a) 1/2 (b) $-x$ (c) $2\log_{10} x$

7 (a) $\dfrac{bcx}{bc-cy-bz}$ (b) $\dfrac{V-2bc}{2(b+c)}$ (c) $\dfrac{-\pi h+\sqrt{\pi^2h^2+2\pi A}}{2\pi}$ (d) $\dfrac{A}{1+nr}$ (e) $\dfrac{2x-y}{x+2y}$ (f) $\dfrac{\pi}{\pi-1}$

8 (a) $y-(-1)=(x-(-2))^2$ (b) $y-\frac{3}{8}=-\frac{3}{2}(x-(-\frac{1}{2}))^2$ (c) $x-(-10)=9(y-\frac{1}{3})^2$

9 (a) $x^4(x-4)(x+4)$ (b) $(x-2)(2x-5)(2x+5)$ (c) $(2x+3)(4x^2-6x+9)$ (d) $(x-1)(x+1)(x^2+1)$

10 (a) $0,\pm4$ (b) $2,\pm\frac{5}{2}$ (c) $-\frac{3}{2}$

11 (a) $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$, or $\frac{11\pi}{6}$ (b) $-\frac{\pi}{2}, \frac{\pi}{6}$, or $\frac{5\pi}{6}$ (c) $\frac{\pi}{6}+2k\pi$, or $\frac{5\pi}{6}+2k\pi$, where k is any integer

12 (a) $-\frac{\sqrt{3}}{2}$ (b) $-\frac{\sqrt{2}}{2}$ (c) $-\frac{\pi}{4}$ (d) $-\frac{\pi}{2}$ (e) $\frac{\sqrt{2}}{2}$ (f) $\frac{\pi}{3}$ (g) $\frac{\sqrt{3}}{3}$ (h) π

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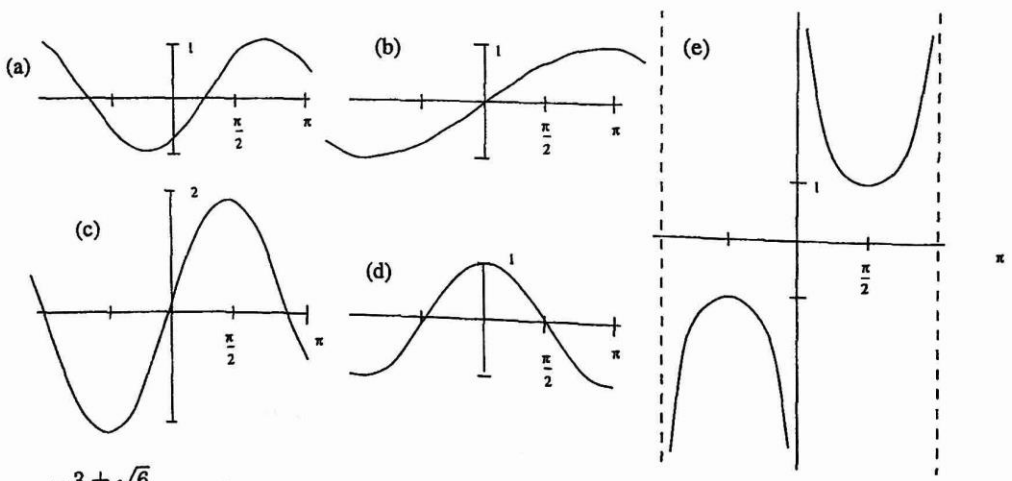


14 (a) $\dfrac{-3\pm\sqrt{6}}{2}$ (b) $\frac{1}{2}$ or -3 (c) $-\frac{1}{2}$

15 (a) -89 (b) x^2+3

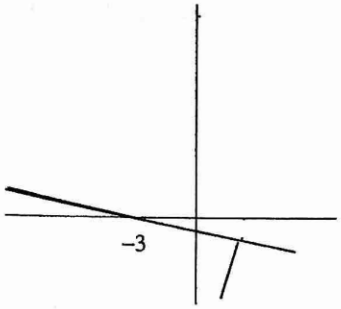
16 (a) $-\frac{1}{3}$ or $\frac{1}{4}$ (b) $-\frac{1}{2}$, $-\frac{1}{2}$, or $\frac{1}{3}$

17 (a) $-3\le x\le 1$ (b) $x<\frac{2}{3}$ or $x\ge 1$ (c) all x

18 (a) $3 \leq x \leq 5$ (b) 2 or $-\frac{6}{5}$ (c) $-\frac{4}{3}$ or 2

19 (a) $7x + 3y = 2$ (b) $3x + 2y = 1$ (c) $y = 3$

20 (a) $(2, -1)$ (b)



21 (a) $(x-1)^2 + (y-2)^2 = 18$ (b) $(x - \frac{1}{2})^2 + (y-1)^2 = \frac{5}{4}$

22 (a) centre $= (-3, 2)$ radius $= \sqrt{10}$ (b) $x + 3y = 13$

23 (a) 9 (b) $(x-5)^2 + (y-3)^2 = 25$

24 $8x^2 - 38x + 8y^2 + 20y + 43 = 0$ (a circle)

25 (a) $x < -2$ or $x > 1$ (b) i D all numbers, R $\{7\}$ ii D all numbers except $-\frac{1}{2}$, R all numbers except $\frac{5}{2}$

26 D all numbers except 0, Range $\{1, -1\}$

27 (a) 2 (b) $\dfrac{-1}{(x+1)(x+h+1)}$ (c) $2x + h$

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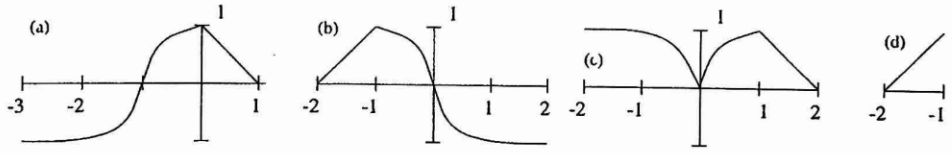

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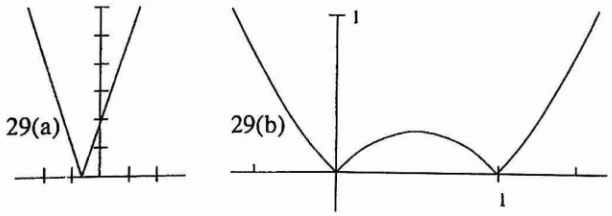
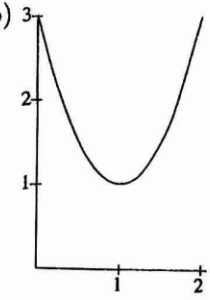


30 (a) $y = -x^2 + 2x + 3$ (b)



31 (a) $y = x^2 - 3x + 2$ (b) $y = x(x^2 + 3x + 3)(x + 1)^3$ (c) $x^2 + y^2 = 1$

32 (a) $f^{-1}(x) = \dfrac{x-3}{2}$ (b) $f^{-1}(x) = \dfrac{x+2}{5x-1}$ (c) $-1 + \sqrt{x+2},\ x > -1$

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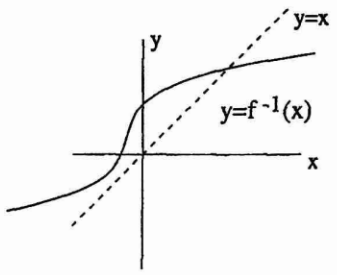

34 (a) $x = t\left(\dfrac{r-h}{h}\right)$ (b) $x = \dfrac{rt}{\sqrt{r^2 - h^2}}$

35 (a) $1 - \frac{\pi}{4}$ (b) $4r + \pi r$ (c) $\frac{9\pi}{4}$ (d) $100\sqrt{5}$ km (e) $\frac{\pi}{6}$ or $30°$

36 (a) Use B, $y = x$, A and $\cos 0 = 1$ (b) Use D (c) Use C (d) Use (c) then (a)
(e) Use (d) then (a) (f) Replace x by $\frac{x}{2}$ in (d) (g) Replace x by $\frac{x}{2}$ in (e)