



Welcome to Algebra 2!

This summer packet is for all students enrolled in Algebra 2 at Herndon High School for Fall 2021.

This summer assignment is not *required*, but it is *strongly recommended*.

The exercises will give you the opportunity to self-assess how prepared you are for Algebra 2. Feel free to use old notes and online resources as needed to ensure that you understand the content.

Tips for summer review:

- Complete the work for this packet on a separate piece of paper.
- Do as many of the problems as you can **WITHOUT the use of a calculator.** It is important to spend time keeping these skills and concepts fresh in your mind – especially your mental math!
- Spend about 20 minutes each day working on the packet.
- You don't have to do the pages in order. Feel free to skip around.
- Keep track of sticky spots ... it's important to know where you need help.
- We will provide you with a key at the start of next year for you to check your work.
- Feel free to reach out to us over the summer. Our contact information:

Mr. Feord cefeord@fcps.edu

Mrs. Drake: mldrake@fcps.edu

Ms. Muse MAMuse@fcps.edu

Mr. Saidi: hssaidi@fcps.edu

FCPS recommended activities for each level of mathematics are also posted on the Herndon High School website. Both resources will help you prepare for next year.

Have a great summer – we are looking forward to meeting you in August!

As you work through the packet, keep track of the following:

“Things I learned, but forgot how to do:”

“Things I never learned:”

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A. Determining Whether a Point is on a Line

Example: Decide whether (3, -2) is a solution of the equation $y = 2x - 8$.

$$\begin{array}{ll} -2 = 2(3) - 8 & \text{Substitute 3 for } x \text{ and } -2 \text{ for } y. \\ -2 = -2 & \text{Simplify} \end{array}$$

The statement is true, so (3, -2) is a solution of the equation $y = 2x - 8$.

Practice: Determine whether each coordinate point is a solution for the given equation.

- | | |
|--------------------------------------|-------------------------------------|
| 1. $y = -10x - 2$; (1, -12) | 3. $9x - y = -4$; (-1, -5) |
| 2. $y = \frac{3}{2}x + 10$; (4, 12) | 4. $y + 5 = \frac{5}{3}x$; (9, 10) |

B. Calculating Slope

Example: Find the slope of a line passing through (3, -9) and (2, -1).

$$\begin{array}{ll} m = \frac{y_2 - y_1}{x_2 - x_1} & \text{Formula for slope} \\ m = \frac{-1 - (-9)}{2 - 3} = \frac{-1 + 9}{-1} & \text{Substitute values and simplify} \\ m = \frac{8}{-1} = -8 & \text{Slope is } -8 \end{array}$$

Practice: Calculate the slope of the line passing through each set of coordinate points.

- | | |
|---------------------|----------------------|
| 5. (5, 6) (9, 8) | 7. (14, -5) (7, 8) |
| 6. (-6, -4) (1, 10) | 8. (-9, 13) (2, -10) |

C. Writing the Equation of a Line

Example 1: Write an equation of the line that passes through the point (3, 4) and has a y-intercept of 5.

$$\begin{array}{ll} y = mx + b & \text{Write the slope-intercept form} \\ 4 = 3m + 5 & \text{Substitute values for } b, x \text{ and } y; \text{ then simplify} \\ -1 = 3m & \\ -\frac{1}{3} = m & \text{Slope is } m = -\frac{1}{3}. \text{ The equation of the line is } y = -\frac{1}{3}x + 5 \end{array}$$

Practice: Write the equation of the line passing through the following point and y intercept.

9. $(-3, 10)$; $b = 8$

11. $(5, -8)$; $b = 7$

10. $(-1, 4)$; $b = -8$

12. $(2, 3)$; $b = 2$

Example 2: Write the equation of the line that passes through the points $(4, 8)$ and $(3, 1)$.

$$m = \frac{1-8}{3-4}$$

Substitute values to find the slope of the line

$$m = \frac{-7}{-1} = 7$$

Simplify.

$$1 = 7(3) + b$$

Substitute values into $y = mx + b$ and solve for b .

$$1 = 21 + b$$

$$-20 = b$$

The equation of the line is $y = 7x - 20$

Practice: Write the equation of the line passing through each set of coordinate points.

13. $(5, -1)$ $(4, -5)$

15. $(-3, 2)$ $(-5, -2)$

14. $(1, 2)$ $(-1, -4)$

16. $(-6, 4)$ $(6, -1)$

D. Distance Formula

Example: Find the distance between the points $(-4, 3)$ and $(-7, 8)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute coordinate values to find the distance

$$= \sqrt{(-7 - (-4))^2 + (8 - 3)^2}$$

Simplify.

$$= \sqrt{(-3)^2 + (5)^2}$$

$$= \sqrt{34}$$

Practice: Find the distance between the following points:

17. $(-3, 4)$ $(1, 4)$

18. $(-8, 5)$ $(-1, 1)$

E. Combining Like Terms

Example: Group like terms and simplify.

$$8x^2 + 16xy - 3x^2 + 3xy - 3x$$

$$8x^2 - 3x^2 + 16xy - 3xy - 3x \quad \text{** Like terms have the same variable to the same power.}$$

$$5x^2 - 3x + 19xy$$

Practice: Simplify.

19. $-5m + 3q + 4m - q$

20. $-3p - 4t - 5t - 2p$

21. $3x^2y - 5xy^2 + 6x^2y$

22. $5x^2 + 2xy - 7x^2 + xy$

F. Solving Equations and Inequalities

Example 1 – Equation with variables on both sides. Solve:

$$6a - 12 = 5a + 9$$

$$a - 12 = 9$$

Subtract $5a$ from each side. Add 12 to each side.

$$a = 21$$

Practice: Solve for the variable.

23. $8m + 1 = 7m - 9$

24. $3a - 12 = -6a - 12$

25. $-7x + 7 = 2x - 11$

Example 2 – Equations with the Distributive Property. Solve:

$$2(5x + 4) = 7x - 3$$

$$10x + 8 = 7x - 3$$

$$3x = -11$$

Distribute 2, solve

$$x = \frac{-11}{3}$$

Practice: Solve for the variable.

26. $-3(4x + 1) = x - 9$

27. $\frac{1}{2}(4m - 6) = 3(m + 5)$

28. $9m + 10 = 3\left(\frac{2}{3}m - 6\right)$

Example 3 – Literal Equations. Solve for x:

$$3x - a = c$$

$$3x = c + a$$

Use properties of equality to isolate the variable.

$$x = \frac{c+a}{3}$$

Practice: Solve for x.

29. $2x + a = bc$

30. $ax + by = c$

31. $2(x + a) = 4b$

Example 4 - Given the formula for the surface area of a right cylinder, solve for h:

$$S = 2\pi r^2 + 2\pi rh$$

or

$$S = 2\pi r(r + h)$$

$$\frac{S}{2\pi} = r + h$$

$$\frac{S}{2\pi} - r = h$$

$$S - 2\pi r^2 = 2\pi rh$$

$$\frac{(S - 2\pi r^2)}{2\pi r} = h$$

Practice: Solve the literal equation for the indicated variable. Assume variables are positive:

32. $A = \frac{1}{2}h(b_1 + b_2)$; b_1

33. $P = 2l + 2w$; l

34. $a^2 + b^2 = c^2$; b

Example 5 – Solving Proportions: Use cross products to solve.

$$\frac{x}{8} = \frac{3}{4}$$

a. $4x = 8 \cdot 3$

$$4x = 24$$

$$x = 6$$

$$\frac{6}{x+4} = \frac{1}{9}$$

b. $6 \cdot 9 = x + 4$

$$54 = x + 4$$

$$50 = x$$

Practice: Use cross products to solve.

35. $\frac{t}{27} = \frac{4}{9}$

36. $\frac{27}{5} = \frac{3}{x}$

37. $\frac{18}{x} = \frac{9}{5}$

38. $\frac{1}{18} = \frac{5}{-4(x-1)}$

39. $\frac{3}{m+4} = \frac{9}{14}$

40. $\frac{r}{3r+1} = \frac{2}{3}$

Example 6 – Inequalities. Solve. Remember when you multiply or divide each side of an inequality by a negative number, you must reverse the inequality symbol to maintain a true statement.

a. $5x - 4 \geq 4x + 6$

$$x - 4 \geq 6$$

$$x \geq 10$$

b. $10 - 7x < 24$

$$-7x < 14$$

$$x > -2$$

Practice: Solve.

41. $-5 + m < 21$

43. $2b + 4 > -3b + 7$

42. $-3x + 4 \leq -5$

44. $14 - 5t \geq 28$

Example 7 – Absolute Value Equations and Inequalities. Solve.

$$|x + 8| = 4$$

a. $x + 8 = 4$ or

$$x + 8 = -4$$

$$x = -4 \text{ and } x = -12$$

$$|x - 5| \geq 20$$

b. $x - 5 \geq 20$ or

$$x - 5 \leq -20$$

$$x \geq 25 \text{ or } x \leq -15$$

$$|x + 1| < 3$$

$$x + 1 < 3 \text{ and}$$

c. $x + 1 > -3$

$$x < 2 \text{ and } x > -4$$

$$-4 < x < 2$$

Practice: Solve.

45. $|5 - x| = 3$

47. $|x - 6| > 8$

46. $|-4x + 5| + 2 = 15$

48. $|9x - 6| - 3 \leq 18$

G. Simplifying Radicals

Example 1: Simplify $\sqrt{20}$:

$$\begin{aligned} \sqrt{20} &= \sqrt{4 \cdot 5} \\ &= 2\sqrt{5} \end{aligned}$$

Simplify: $\sqrt{\frac{25}{9}}$

$$\begin{aligned} \sqrt{\frac{25}{9}} &= \frac{\sqrt{25}}{\sqrt{9}} \\ &= \frac{5}{3} \end{aligned}$$

Simplify: $\frac{\sqrt{32}}{\sqrt{50}}$

$$\begin{aligned} \frac{\sqrt{32}}{\sqrt{50}} &= \sqrt{\frac{32}{50}} \\ &= \sqrt{\frac{16}{25}} = \frac{\sqrt{16}}{\sqrt{25}} \\ &= \frac{4}{5} \end{aligned}$$

Helpful hints with radicals: a radical is in simplest form if there are 1) no fractions in the radicand, 2) no perfect squares in the radicand and 3) no radicals in the denominator. LOOK for ways to simplify the fraction BEFORE you rationalize the denominator!

Practice: Simplify:

49. $\sqrt{12}$

50. $\sqrt{18}$

51. $\sqrt{40}$

52. $\sqrt{243}$

53. $\sqrt{320}$

54. $\frac{\sqrt{12}}{\sqrt{3}}$

55. $\sqrt{\frac{27}{4}}$

Example 2 - Operations with Radicals. Radicals must have same radicand to add or subtract.

a. $5\sqrt{3} - \sqrt{3} - \sqrt{2}$
 $= 4\sqrt{3} - \sqrt{2}$

b. $(2\sqrt{2})(5\sqrt{3})$
 $= 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3}$
 $= 10\sqrt{6}$

c. $(5\sqrt{7})^2$
 $= 5^2 \cdot \sqrt{7^2}$
 $= 25 \cdot 7$
 $= 175$

Practice: Solve.

56. $\sqrt{27} - \sqrt{12}$

57. $\sqrt{64} - \sqrt{28}$

58. $\sqrt{242} + \sqrt{200}$

59. $\sqrt{20} + \sqrt{45} - \sqrt{5}$

60. $\sqrt{363} \cdot \sqrt{300}$

61. $\sqrt{21} \cdot \sqrt{24}$

62. $(5\sqrt{4})(2\sqrt{4})$

63. $(3\sqrt{3})^2$

64. $(10\sqrt{11})^2$

Example 3 – Rationalizing the denominator. Simplify the quotient $\frac{6}{\sqrt{5}}$

$$\begin{aligned} \frac{6}{\sqrt{5}} &= \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}} \\ &= \frac{6\sqrt{5}}{5} \end{aligned}$$

Multiply the numerator and denominator by $\sqrt{5}$ to eliminate the radical in the denominator

Practice: Simplify. Rationalize all denominators

65. $\frac{5}{\sqrt{2}}$

66. $\frac{16}{\sqrt{24}}$

67. $\frac{9}{\sqrt{52}}$

68. $\frac{\sqrt{27}}{\sqrt{45}}$

69. $\frac{\sqrt{50}}{\sqrt{75}}$

H. Properties of Exponents

Example: An expression like 5^3 is called a **power**. The **exponent**³ represents the number of times the **base** (5) is used as a factor: $5^3 = 5 \cdot 5 \cdot 5$ (3 factors of 5). To simplify expressions involving exponents, you must use the following properties of exponents:

CONCEPT SUMMARY	MULTIPLICATION PROPERTIES OF EXPONENTS
Let a and b be numbers and let m and n be positive integers.	
PRODUCT OF POWERS PROPERTY	
To multiply powers having the same base, add the exponents.	
$a^m \cdot a^n = a^{m+n}$	Example: $3^2 \cdot 3^7 = 3^{2+7} = 3^9$
POWER OF A POWER PROPERTY	
To find a power of a power, multiply the exponents.	
$(a^m)^n = a^{m \cdot n}$	Example: $(5^2)^4 = 5^{2 \cdot 4} = 5^8$
POWER OF A PRODUCT PROPERTY	
To find a power of a product, find the power of each factor and multiply.	
$(a \cdot b)^m = a^m \cdot b^m$	Example: $(2 \cdot 3)^6 = 2^6 \cdot 3^6$

$$\begin{aligned} \text{a: } & (-3xy^2)^3 \cdot y \\ & = (-3)^3 \cdot x^3 \cdot (y^2)^3 \cdot y^1 \\ & = -27x^3 \cdot y^{6+1} \\ & = -27x^3y^7 \end{aligned}$$

$$\begin{aligned} \text{b: } & \frac{1}{r^7} \cdot r^4 \\ & = \frac{r^4}{r^7} \\ & = r^{4-7} \\ & = r^{-3} \\ & = \frac{1}{r^3} \end{aligned}$$

DEFINITION OF ZERO AND NEGATIVE EXPONENTS
Let a be a nonzero number and let n be a positive integer.
• A nonzero number to the zero power is 1: $a^0 = 1, a \neq 0$.
• a^{-n} is the reciprocal of a^n : $a^{-n} = \frac{1}{a^n}, a \neq 0$.

CONCEPT SUMMARY	DIVISION PROPERTIES OF EXPONENTS
Let a and b be numbers and let m and n be integers.	
QUOTIENT OF POWERS PROPERTY	
To divide powers having the same base, subtract exponents.	
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	Example: $\frac{3^7}{3^5} = 3^{7-5} = 3^2$
POWER OF A QUOTIENT PROPERTY	
To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.	
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	Example: $\left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$

$$\begin{aligned} \text{c: } & \frac{1}{x^6} \cdot \left(\frac{x}{2}\right)^6 \\ & = \frac{1}{x^6} \cdot \frac{x^6}{2^6} \\ & = \frac{x^6}{64x^6} \\ & = \frac{1}{64} \end{aligned}$$

Practice: Simply:

70. $(2ab)^5$

71. $2n^4(3n)^2$

72. $(8^{-1})^{-3}$

73. $\left(\frac{5}{m}\right)^3$

74. $(5x \cdot x^3)^4$

75. $\left(\frac{x^4}{x^3}\right)^2$

76. $(3c^{-4}d^5)^{-2}(cd^{-4})$

77. $\left(\frac{2x^0}{8y^{-7}}\right)$

78. $4y^3z \cdot \left(\frac{y}{2z}\right)^{-3}$

I. Solving Systems of Equations

Example 1: Use Substitution to solve the linear system:

$$3x + 2y = 16 \text{ equation 1}$$

$$x + 3y = 10 \text{ equation 2}$$

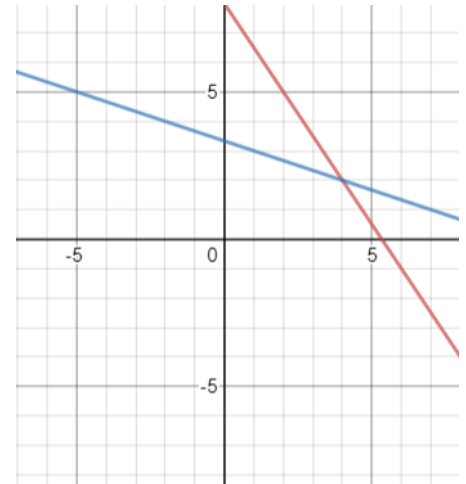
Solve for x in equation 2 since it is easy to isolate x : $x = 10 - 3y$

Substitute $(10 - 3y)$ for x in Equation 1: $3(10 - 3y) + 2y = 16$.

Solve for y to get $y = 2$.

Substitute value for y into the equation: $x = 10 - 3(2)$
 $x = 4$

The solution is $(4, 2)$, the ordered pair that makes BOTH equations true.



CHECK:

- substitute 4 for x and 2 for y in the original equations.
- graph the original equations in the same coordinate plane. the graphs should intersect at $(4, 2)$.

Practice: Use Substitution to solve the system of linear equations.

$$79. \begin{cases} 2x - 3y = -16 \\ y = 5x + 1 \end{cases}$$

$$80. \begin{cases} 3x + y = 6 \\ 5(x + y) = 20 \end{cases}$$

Example 2: Use Linear Combinations to solve the linear system: $\begin{cases} 4x - 3y = -5 \\ 7x + 2y = -16 \end{cases}$

The goal is to obtain coefficients that are opposites for one of the variables.

$$4x - 3y = -5 \quad \text{multiply by 2} \quad \Rightarrow \quad 8x - 6y = -10$$

$$7x + 2y = -16 \quad \text{multiply by 3} \quad \Rightarrow \quad + \underline{21x + 6y = -48}$$

$$29x = -58$$

$$x = -2$$

Add the equations

Solve for x

- **Substitute** -2 for x : $4(-2) - 3y = -5$.
- **Solve** to get $y = -1$
- The solution is $(-2, -1)$.
- **Check** the ordered pair in the original equations.

Practice: Use Linear Combinations to solve the system of linear equations.

$$81. \begin{cases} 6x + 2y = 13 \\ 4x + y = 11 \end{cases}$$

$$82. \begin{cases} x = \frac{1}{2}y + 3 \\ 2x - y = 3 \end{cases}$$

Remember the special case scenarios ... parallel lines do not intersect and coincident lines have infinite solutions!

J. Polynomials

Example 1: Find the product and simplify: $(x+4)(x+3)$

When multiplying polynomials, use the distributive property to make sure each term in the expression is multiplied by the others. Combine Like Terms if you can.

“FOIL” (First, Inner, Outer, Last) is a trick to help you remember each part if the terms are Binomials.

Distributive Property:

$$\begin{aligned} &= (x+4)(x+3) \\ &= (x+4)x + (x+4)3 \\ &= x^2 + 4x + 3x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

FOIL: (first, outer, inner, last)

$$\begin{aligned} &= (x+4)(x+3) \\ &= x^2 + 3x + 4x + 12 \\ &= x^2 + 7x + 12 \end{aligned}$$

Watch out for special patterns and trinomial factors!

Practice: Find the product:

83. $(x+2)(x-6)$

84. $(x-1)^2$

85. $(15-y)(3+y)$

86. $(x+7)(x^2+2x-3)$

87. $(3x+5)(3x-5)$

88. $(x^2-2x+1)(x+4)$

89. $(2x-y)(2x+y)$

90. $(x^2-1)(x^2+8)$

Example 2 – Factoring

- To **factor** a quadratic expression means to write it as the product of two linear expressions.

- To factor $x^2 + bx + c$, you need to find numbers p and q such that:

$$\mathbf{p + q = b \quad \text{and} \quad pq = c}$$

- Remember, $x^2 + bx + c = (x + p)(x + q)$ when $p + q = b$ and $pq = c$

Practice: Factor the Trinomial

91. $x^2 + 5x + 6$

92. $x^2 + 6x + 5$

93. $x^2 - 5x + 6$

94. $x^2 - 3x + 2$

95. $x^2 - 7x + 12$

96. $8x^2 - 2x - 3$

97. $3x^2 + 13x - 14$

98. $5x^2 + 27x - 18$

99. $6x^2 - 7x + 1$

K. Quadratics

A **quadratic equation** is an equation that can be written in the standard form: $ax^2 + bx + c = 0$ where $a \neq 0$.

Example 1: Solving $ax^2 + c = 0$

When $b = 0$, the quadratic equation has the form $ax^2 + c = 0$. **In this case you can isolate the radical and solve for x .** Remember when taking the square root to solve for the positive and the negative root.

a. $3x^2 - 1 = 23$
 $3x^2 = 24$
 $x^2 = 8$
 $x = \pm\sqrt{8}$
 $x = \pm 2\sqrt{2}$

b. $12 - x^2 = 13$
 $-x^2 = 1$
 $x^2 = -1$
no real solution

c. $4 + 2n^2 = 4$
 $2n^2 = 0$
 $n^2 = 0$
 $n = 0$

Practice: Solve for x :

100. $x^2 = 289$
101. $x^2 - 7 = 6$

102. $6x^2 = 294$
103. $9x^2 + 7 = 52$

Example 2: Using the Quadratic Formula

Formula to find the solutions of a quadratic equation $ax^2 + bx + c = 0$ are: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Solve $x^2 - 4x - 12 = 0$ by using the quadratic formula.

Substitute $a = 1$, $b = -4$, $c = 12$ into the quadratic formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2}$$
$$x = \frac{4 \pm \sqrt{64}}{2}$$

The solutions are: **6 and -2.** (Check by substituting into the equation.)

Practice: Use the quadratic formula to solve each equation. Round solutions to the nearest 100th.

104. $a^2 + 8 = 6a$

105. $-25 = x^2 + 10x - 5$

106. $4x^2 - 3x = 7$

Whew! That was a lot of work, wasn't it? One more task for you:

Click here for a google form to check your answers: [Solutions](#)

Keep track of any questions you have or specific topics you need to review!