

A. Calculating Slope

Example: Find the slope of a line passing through (3, -9) and (2, -1).

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Formula for slope

$$m = \frac{-1 - (-9)}{2 - 3} = \frac{-1 + 9}{-1}$$

Substitute values and simplify

$$m = \frac{8}{-1} = -8$$

Slope is -8

Note: for each point (x, y) in the fraction to calculate slope, the y value must be directly above the x value of the same point.

Exercises: Calculate the slope of the line passing through each set of coordinate points.

1. (5, 6) (9, 8) $m = \frac{8 - 6}{9 - 5} = \frac{2}{4} = \frac{1}{2}$

$$m = \frac{1}{2}$$

3. (14, -5) (7, 8) $m = \frac{-5 - 8}{14 - 7} = \frac{-13}{7}$

$$m = \frac{-13}{7}$$

2. (-6, -4) (1, 10) $m = \frac{10 - (-4)}{1 - (-6)} = \frac{14}{7} = \frac{2}{1}$

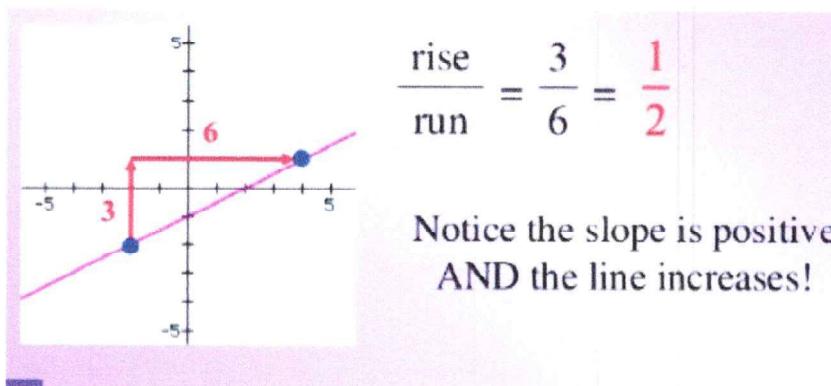
$$m = 2$$

4. (-9, 13) (2, -10) $m = \frac{13 - (-10)}{-9 - 2} = \frac{23}{-11}$

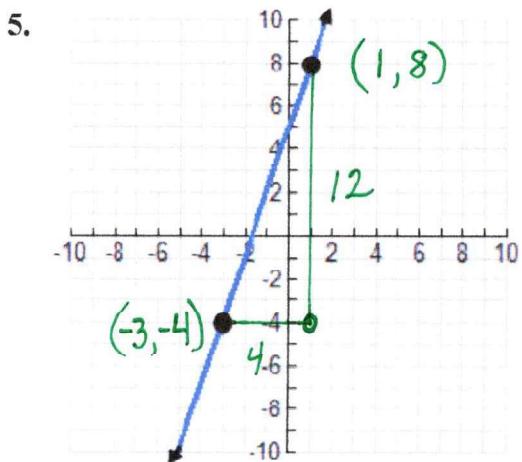
$$m = -\frac{23}{11}$$

$$m = \frac{23}{-11}$$

B. Given the graph, find the slope of the line

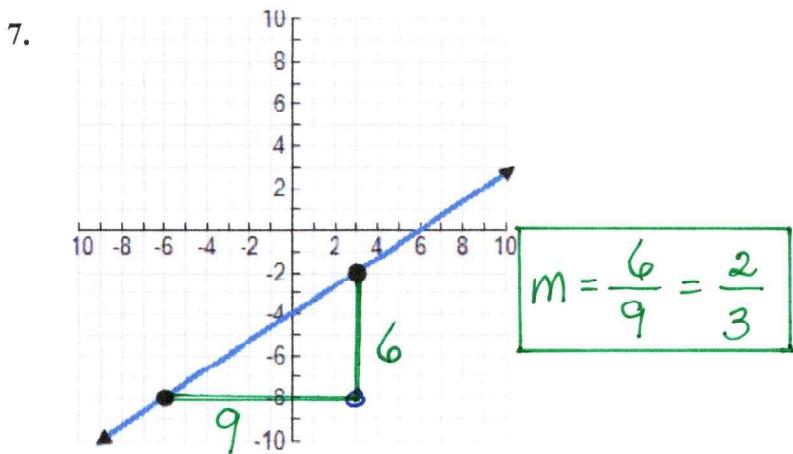
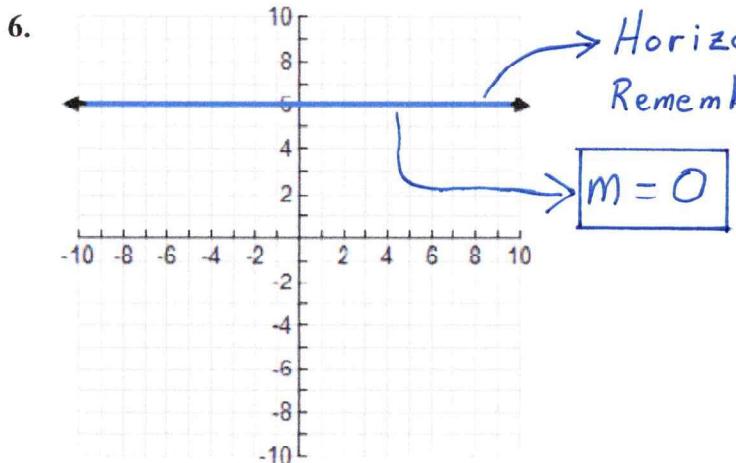


Notice the slope is positive
AND the line increases!



$$m = \frac{12}{4} = \frac{3}{1} = 3 \} \text{ By counting spaces on graph.}$$

$$m = \frac{8 - (-4)}{1 - (-3)} = \frac{12}{4} = 3 \} \text{ By using } x, y \text{ values from 2 points.}$$



C. Writing the Equation of a Line

Example: Write an equation of the line that passes through the point $(3, 4)$ and has a y-intercept of 5 .

$$y = mx + b$$

Write the slope-intercept form

$$4 = 3m + 5$$

Substitute values for b , x and y ; then simplify

$$-1 = 3m$$

$$-\frac{1}{3} = m$$

Slope is $m = -\frac{1}{3}$. The equation of the line is $y = -\frac{1}{3}x + 5$

Exercises: Write the equation of the line passing through the following point and y intercept.

8. $(-3, 10)$; $b = 8$

$$\begin{aligned} y &= mx + b \\ 10 &= m(-3) + 8 \\ 2 &= -3m \\ \frac{2}{-3} &= m \end{aligned}$$

9. $(-1, 4)$; $b = -8$

$$\begin{aligned} y &= mx + b \\ 4 &= m(-1) + (-8) \\ 12 &= -m \\ -12 &= m \end{aligned}$$

10. $(5, -8)$; $b = 7$

$$y = -3x + 7$$

$$\begin{aligned} -8 &= m(5) + 7 \\ -15 &= 5m \\ -3 &= m \end{aligned}$$

11. $(2, 3)$; $b = 2$

$$y = \frac{1}{2}x + 2$$

$$\begin{aligned} 3 &= m(2) + 2 \\ 1 &= 2m \\ \frac{1}{2} &= m \end{aligned}$$

D. Writing the Equation of a Line

Example: Write the equation of the line that passes through the points (4, 8) and (3, 1).

$$m = \frac{1-8}{3-4}$$

Substitute values to find the slope of the line

$$m = \frac{-7}{-1} = 7$$

Simplify.

$$1 = 7(3) + b$$

$$1 = 21 + b$$

$$-20 = b$$

Substitute values into $y = mx + b$ and solve for b .

The equation of the line is $y = 7x - 20$

Exercises: Write the equation of the line passing through each set of coordinate points.

12. (5, -1) (4, -5)

$$m = \frac{-1 - (-5)}{5 - 4} = \frac{4}{1} = 4$$

$$y = 4x - 19$$

14. (-3, 2) (-5, -2)

$$m = \frac{2 - (-2)}{-3 - (-5)} = \frac{4}{2} = 2$$

$$y = 2x + 8$$

13. (1, 2) (-1, -4)

$$m = \frac{2 - (-4)}{1 - (-1)} = \frac{6}{2} = 3$$

$$y = 3x - 1$$

$$2 = 3(1) + b$$

$$-4 = 3(-1) + b$$

$$-1 = b$$

15. (-6, 4) (6, -1)

$$m = \frac{4 - (-1)}{-6 - 6} = \frac{5}{-12} = -\frac{5}{12}$$

$$y = -\frac{5}{12}x + 9$$

$$4 = -\frac{5}{12}(12) + b$$

$$4 = -5 + b$$

$$9 = b$$

E. Distance Formula

Example: Find the distance between the points (-4, 3) and (-7, 8).

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute coordinate values to find the distance

$$= \sqrt{(-7 - (-4))^2 + (8 - 3)^2}$$

Simplify.

$$= \sqrt{(-3)^2 + (5)^2}$$

$$= \sqrt{34}$$

Exercises: Find the distance between the following points:

16. (-3, 4) (1, 4)

$$d = \sqrt{(-3 - 1)^2 + (4 - 4)^2}$$

$$d = \sqrt{(-4)^2 + (0)^2}$$

$$d = \sqrt{16}$$

$$d = 4$$

17. (-8, 5) (-1, 1)

$$d = \sqrt{(-8 - (-1))^2 + (5 - 1)^2}$$

$$d = \sqrt{(-7)^2 + (4)^2}$$

$$d = \sqrt{49 + 16} = \sqrt{65}$$

$$d = \sqrt{65}$$

F. Combining Like Terms**Example:** Simplify.

(Note: Terms are defined by their variable(s) and exponent(s), terms are separated by + or -.)

Group like terms and simplify.

$$8x^2 + 16xy - 3x^2 + 3xy - 3x$$

$$8x^2 - 3x^2 + 16xy - 3xy - 3x$$

$$5x^2 - 3x + 19xy$$

Exercises: Simplify.

$$18. -5m + 3q + 4m - q = -5m + 4m + 3q - q \\ = -m + 2q$$

$$19. -3p - 4t - 5t - 2p \\ -3p - 2p - 4t - 5t = -5p - 9t$$

$$20. 3x^2y - 5xy^2 + 6x^2y = 3x^2y + 6x^2y - 5xy^2 \\ 9x^2y - 5xy^2$$

$$21. 5x^2 + 2xy - 7x^2 + xy = 5x^2 - 7x^2 + 1xy + xy \\ -2x^2 + 3xy$$

G. Solving Equations With Variables on Both Sides**Example:** Solve.

$$6a - 12 = 5a + 9$$

$$a - 12 = 9$$

$$a = 21$$

Subtract $5a$ from each side. Add 12 to each side.**Exercises:** Simplify.

$$22. 8m + 1 = 7m - 9 \\ 8m - 7m = -9 - 1 \\ m = -10$$

$$23. 3a - 12 = -6a - 12 \\ 3a + 6a = -12 + 12 \\ 9a = 0 \\ a = 0$$

$$24. -7x + 7 = 2x - 11 \\ 7 + 11 = 2x + 7x \\ 18 = 9x \\ 2 = x$$

$$26. 16 = \frac{3}{4}x + 1 \\ 16 - 1 = \frac{3}{4}x \\ \frac{4}{3} \cdot (15) = x \\ 20 = x$$

$$27. -(4x - 8) = 2(x + 4) \\ -4x + 8 = 2x + 8 \\ 8 + 8 = 2x + 4x \\ 16 = 6x \\ \frac{16}{6} = x \\ \frac{8}{3} = x$$

$$28. \frac{1}{2}(x - 16) = 7 \\ (x - 16) = 7 \cdot \frac{2}{1} = 14 \\ x = 14 + 16 \\ x = 30$$

H. Solving Proportions

Example: Use cross products to solve.

$$\frac{x}{8} = \frac{3}{4}$$

a. $4x = 8 \cdot 3$
 $4x = 24$
 $x = 6$

$$\frac{6}{x+4} = \frac{2}{9}$$

$$6 \cdot 9 = 2(x+4)$$

b. $54 = 2x + 8$
 $46 = 2x$
 $23 = x$

Exercises: Use cross products to solve.

29. $\frac{t}{27} = \frac{4}{9}$ $9t = 4 \cdot 27$
 $t = 12$

30. $\frac{27}{5} = \frac{3}{x}$ $27x = 3 \cdot 5$
 $x = \frac{15}{27}$
 $x = \frac{5}{9}$

31. $\frac{19}{x} = \frac{9}{5}$
 $9x = 19 \cdot 5$
 $x = \frac{95}{9}$

32. $\frac{1}{18} = \frac{5}{-4(x-1)}$
 $1 \cdot (-4)(x-1) = 5 \cdot 18$
 $-4x + 4 = 90$
 $-4x = 86$
 $x = \frac{-43}{2}$

33. $\frac{3}{m+4} = \frac{9}{14}$
 $m+4 = \frac{14}{3}$
 $m = \frac{2}{3}$

34. $\frac{r}{3r+1} = \frac{2}{3}$
 $r = \frac{-2}{3}$

$$3 \cdot 14 = 9(m+4)$$

$$42 = 9m + 36$$

$$6 = 9m$$

$$3r = 2(3r+1)$$

$$3r = 6r + 2$$

$$-2 = 3r$$

I. Simplifying Radicals

Example: Simplify the expression $\sqrt{20}$

$$\begin{aligned}\sqrt{20} &= \sqrt{4 \cdot 5} \\ &= 2\sqrt{5}\end{aligned}$$

Use Product Property to simplify.

Exercises: Solve:

35. $\sqrt{52} = \sqrt{4} \cdot \sqrt{13}$
 $= 2\sqrt{13}$

37. $\sqrt{243} = \sqrt{81} \cdot \sqrt{3}$
 $= 9\sqrt{3}$

36. $\sqrt{40} = \sqrt{4} \cdot \sqrt{10}$
 $= 2\sqrt{10}$

38. $\sqrt{320} = \sqrt{5} \cdot \sqrt{64}$
 $= 8\sqrt{5}$

J. Simplifying Radical Expressions

Example: Simplify the radical expression.

$$5\sqrt{3} - \sqrt{3} - \sqrt{2}$$

a. $= 4\sqrt{3} - \sqrt{2}$

$$(2\sqrt{2})(5\sqrt{3})$$

b. $= 2 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{3}$
 $= 10\sqrt{6}$

$$\begin{aligned} & (5\sqrt{7})^2 \\ & c. = 5^2 \sqrt{7^2} \\ & = 25 \cdot 7 \\ & = 175 \end{aligned}$$

Exercises: Solve.

39. $\sqrt{64} - \sqrt{28} = \boxed{8 - 2\sqrt{7}}$

43. $\sqrt{21} \cdot \sqrt{24} = \boxed{\sqrt{3 \cdot 3 \cdot 2 \cdot 2 \cdot 2 \cdot 7}} \\ = 6\sqrt{14}$

40. $\sqrt{242} + \sqrt{200} = 11\sqrt{2} + 10\sqrt{2}$
 $= \boxed{21\sqrt{2}}$

44. $(5\sqrt{4})(2\sqrt{4}) = 5 \cdot 2 \cdot \sqrt{4} \cdot \sqrt{4}$
 $= 10 \cdot 4 = \boxed{40}$

41. $\sqrt{20} + \sqrt{45} - \sqrt{5} = \sqrt{4} \cdot \sqrt{5} + \sqrt{9} \cdot \sqrt{5} - \sqrt{5}$
 $= \boxed{4\sqrt{5}}$
 $= (2+3-1) \cdot \sqrt{5}$

45. $(8\sqrt{3})^2 = 8^2 \cdot (\sqrt{3})^2$
 $= 64 \cdot 3 = \boxed{192}$

42. $\sqrt{363} \cdot \sqrt{300} = 11\sqrt{3} \cdot 10\sqrt{3}$
 $= \boxed{330}$
 $= 11 \cdot 10 \cdot 3$

46. $(10\sqrt{11})^2 = 100 \cdot 11 = \boxed{1,100}$

K. Simplifying Quotients with Radicals

Example: Simplify the quotient $\frac{6}{\sqrt{5}}$

$$\begin{aligned} \frac{6}{\sqrt{5}} &= \frac{6}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{6\sqrt{5}}{\sqrt{5}\sqrt{5}} \\ &= \frac{6\sqrt{5}}{5} \end{aligned}$$

Multiply the numerator and denominator by $\sqrt{5}$ to eliminate
 the radical in the denominator

Exercises: Solve:

47. $\frac{16}{\sqrt{24}} = \frac{16}{2\sqrt{6}} = \frac{8}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{8\sqrt{6}}{6}$
 $= \boxed{\frac{4\sqrt{6}}{3}}$

49. $\frac{\sqrt{27}}{\sqrt{45}} = \frac{\sqrt{3} \cdot \cancel{\sqrt{9}}}{\sqrt{5} \cdot \cancel{\sqrt{9}}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{\sqrt{15}}{5}}$

48. $\frac{9}{\sqrt{52}} = \frac{9}{2\sqrt{13}} \cdot \frac{\sqrt{13}}{\sqrt{13}} = \boxed{\frac{9\sqrt{13}}{26}}$

50. $\frac{\sqrt{50}}{\sqrt{75}} = \frac{\sqrt{2} \cdot \cancel{\sqrt{25}}}{\sqrt{3} \cdot \cancel{\sqrt{25}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{6}}{3}}$

Helpful hints with radicals: a radical is in simplest form if there are 1) no fractions in the radicand, 2) no perfect squares in the radicand and 3) no radicals in the denominator. LOOK for ways to simplify the fraction BEFORE you rationalize the denominator!

L. Solving Literal Equations

Example: Given the formula for the surface area of a right cylinder, solve for h :

$$S = 2\pi r^2 + 2\pi r h$$

or

$$S = 2\pi r(r + h)$$

$$\frac{S}{2\pi} = r + h$$

$$\frac{S}{2\pi r} - r = h$$

$$S - 2\pi r^2 = 2\pi r h$$

$$\frac{(S - 2\pi r^2)}{2\pi r} = h$$

Exercises: Solve the literal equation for the indicated variable. Assume variables are positive:

51. $A = \frac{1}{2}h(b_1 + b_2)$; b_1

$$2A = h(b_1 + b_2)$$

$$\frac{2A}{h} = b_1 + b_2$$

$$\boxed{\frac{2A}{h} - b_2 = b_1}$$

52. $P = 2l + 2w$; l

$$P - 2w = 2l$$

$$\boxed{\frac{P - 2w}{2} = l}$$

53. $a^2 + b^2 = c^2$; b

$$b^2 = c^2 - a^2$$

$$b = \sqrt{c^2 - a^2}$$